

On $d = 4, 5, 6$ Vacua with 8 Supercharges

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Abstract

We show how $N = 2, d = 4, 5, 6$ maximally supersymmetric vacua (Hpp -waves and $aDS \times S$ solutions) are related through dimensional reduction/oxidation preserving all the unbroken supersymmetries. In particular we show how the $N = 2, d = 5$ family of vacua (which are the near-horizon geometry of supersymmetric rotating black holes) interpolates between $aDS_2 \times S^3$ and $aDS_3 \times S^2$ in parameter space and how it can be dimensionally reduced to an $N = 2, d = 4$ dyonic Robinson-Bertotti solution with geometry $aDS_2 \times S^2$ and oxidized to an $N = 2, d = 6$ solution with $aDS_3 \times S^3$ geometry (which is the near-horizon limit of the self-dual string).

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Introduction

There is currently a renewed interest on maximally supersymmetric vacua stemming from the discovery, and re-discovery of previously overlooked, maximally supersymmetric Hpp-wave solutions [1, 2, 3]. These solutions have very interesting properties: they are not only supergravity solutions (i.e. solution of the lowest-order superstring effective action) but it can be argued that they are exact solutions of superstring theory to all orders and therefore good vacua on which superstrings can be quantized [4, 5] and the D-branes can be discussed [6, 7]. Further, these solutions can be obtained by a limiting procedure that preserves (or increases) the number of unbroken (super)symmetries [8, 9, 10] (for a review see, e.g. Ref. [11]), a feature which has given rise to the Hpp/CFT correspondence (See e.g. [12]).

It is the standard lore that maximally supersymmetric vacua (other than products of Minkowski spacetime by circles) of higher-dimensional supergravity theories cannot be dimensionally reduced preserving all their unbroken supersymmetries (See e.g. [13, 14] and references therein): in general, the Killing spinors of these vacua depend on all coordinates. This dependence complicates its compactification and dimensional reduction. First, only for certain radii of the compact direction the Killing spinors will have the right periodicity and, thus, only for those radii the compactified solutions preserve the same amount of supersymmetry as the uncompactified one. Second, unless the Killing spinors are independent of the compact coordinates (or have a very special dependence on them, as in some generalized dimensional reductions [15]), the components of the Killing spinor that do depend on the compact coordinate have to be projected out of the dimensionally reduced theory [16], leading to less supersymmetry. Since T duality of classical solutions involves their dimensional reduction it should not come as a surprise that the supersymmetry of the maximally supersymmetric vacua is not preserved by T duality either [17, 18].

In this paper we are going to show that the maximally supersymmetric $d = 4, 5, 6$ vacua of theories with 8 supercharges ($N = 2$ or $N = (2, 0)$ theories) can be dimensionally reduced/oxidized preserving all their unbroken supersymmetries because in all the $d = 5, 6$ cases it is possible to choose coordinates in which the Killing spinor is independent of the coordinate we use for dimensional reduction.

That the coordinate choice that preserves all supersymmetry in dimensional reduction is always possible looks highly non-trivial. However, thinking in terms of oxidation of the $d = 4, 5$ theories it is evident that all unbroken supersymmetry should be preserved: these theories can be obtained by standard dimensional reduction of the $d = 6, 5$ ones supplemented by a truncation of the matter multiplets that appear in the reduction. It is, therefore, guaranteed that, if we have a solution of the $d = 4, 5$ theories that preserves all 8 supersymmetries, it comes from some $d = 5, 6$ solution that also preserves those 8 supersymmetries and therefore has to be one of the essentially unique maximally supersymmetric vacua of the theory. Thus, the maximally supersymmetric vacua of these theories must be related by dimensional reduction/oxidation and we are going to show exactly how this happens. The independence of the Killing spinors of the compact coordinates is an implicit automatic consequence of the above arguments.

Let us now briefly review the known maximally supersymmetric vacua of these theories:

$N = (2, 0), d = 6$:

1. The 1-parameter family of Kowalski-Glikman (KG) Hpp-wave solutions found

in Ref. [19] that we will denote by $KG6(2, 0)$.

2. The 1-parameter family of solutions with $aDS_3 \times S^3$ geometry found in Ref. [20] as the near-horizon limit of the self-dual string solution.

$N = 2, d = 5$:

1. The 1-parameter family of KG solutions found in Ref. [19] that we will denote by $KG5$.
2. The 1-parameter family of solutions with $aDS_3 \times S^2$ geometry found in Ref. [20] as near-horizon limit of the extreme string solution.
3. The 1-parameter family of solutions with $aDS_2 \times S^3$ geometry found in Ref. [21] as near-horizon limit of the extreme black hole solution.
4. The 2-parameter family of $N = 2, d = 5$ solutions found in Ref. [22] as the near-horizon limit of the supersymmetric rotating black hole solution.

The third family is contained in the fourth and corresponds to a vanishing rotation parameter. We will show that the second family is also contained in the fourth and corresponds to the value 1 of the rotation parameter.

$N = 2, d = 4$:

1. The 1-parameter⁴ family of KG solutions found in Ref. [23] that we will denote by $KG4$.
2. The 2-parameter family of electric/magnetic $N = 2, d = 4$ Robinson-Bertotti solutions [24] that have the geometry $aDS_2 \times S^2$.

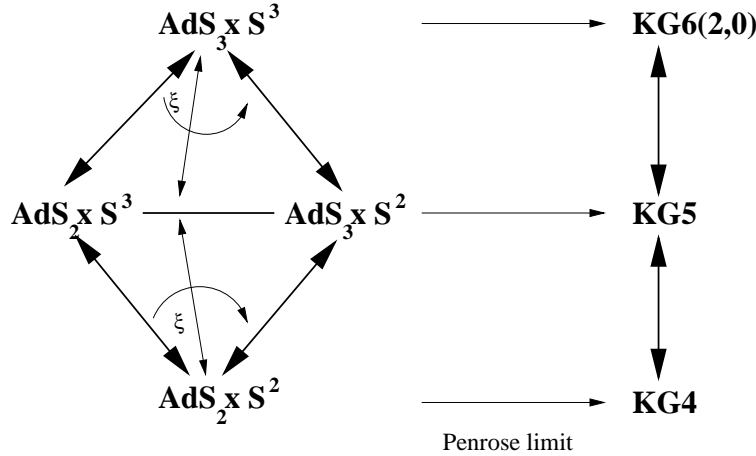


Figure 1: Relations between the $d = 4, 5, 6$ vacua with 8 supercharges.

The connections between these vacua that we have found are summarized in Figure 1. The relations between the KG solutions are straightforward. The $aDS_3 \times S^3$ can be

⁴Electric-magnetic duality rotations only change the polarization plane of an electromagnetic wave and their effect on this family of solutions can be undone by a rotation that leaves the form of the metric invariant.

dimensionally reduced in the direction of the S^1 Hopf fiber of the 3-sphere and then we get $aDS_3 \times S^2$. It can also be reduced in the S^1 fiber of the aDS_3 , giving $aDS_2 \times S^3$. Finally, we can rotate these fibers an angle ξ and reduce, getting the maximally supersymmetric solution that is the near-horizon limit of the rotating 5-dimensional extreme black hole. The angular momentum parameter j is essentially $\sin \xi$. Thus, this 2-parameter family of 5-dimensional vacua interpolates (in parameter space) between the $aDS_2 \times S^3$ and the $aDS_3 \times S^2$ vacua. The reduction of any member of this family in the remaining fiber gives an electric/magnetic Robinson-Bertotti solution where $\sin \xi$ is the quotient between the electric and the magnetic fields.

This paper is organized as follows: in Section 1 we study how the $KG6(2,0)$ and $KG5$ solutions can be dimensionally reduced preserving all the supersymmetry after describing briefly the general form of pp -wave solutions and their sources in Section 1.1. In Section 2 we study how the $aDS_m \times S^n$ -type vacua of these theories are related by oxidizing them. Section 3 contains our conclusions and some discussion.

1 Dimensional Reduction of Maximally Supersymmetric Hpp -Waves

Before we study the reduction of KG solutions it is worth studying briefly general supergravity pp -wave solutions.

1.1 General pp -Wave Solutions

pp -waves spacetimes are those whose metric admits a covariantly constant null vector. A metric with this property can always be put in the form

$$ds^2 = 2du(dv + Kdu + \mathcal{A}_a dx^a) + \tilde{g}_{ab} dx^a dx^b, \quad (1.1)$$

where the functions $K, \mathcal{A}_a, \tilde{g}_{ab}$ depend only on the wave-front coordinates x^a and on the null coordinate u . \mathcal{A}_a is known as the *Sagnac connection* and can always be set to zero by means of a coordinate transformation.

In supergravity theories it is natural to look for pp -wave solutions of the system

$$S_a = \frac{1}{16\pi G_N^{(d)}} \int d^d x \sqrt{|g|} \left[R + \frac{1}{2}(\partial\varphi)^2 + \frac{(-1)^{p+1}}{2(p+2)!} e^{-2a\varphi} F_{(p+2)}^2 \right], \quad (1.2)$$

where $F_{(p+2)} = dA_{(p+1)}$, of the form

$$\begin{cases} ds^2 &= 2du(dv + Kdu) + \tilde{g}_{ab} dx^a dx^b, \\ F_{(p+2)} &= du \wedge C, \end{cases} \quad (1.3)$$

where C is a $(p+1)$ -form on the wave-front space and, as all the other fields in this Ansatz, it is independent of v .

A general solution is provided by a Ricci-flat wave-front metric \tilde{g}_{ab} which must also satisfy

$$\tilde{\nabla}_a (\tilde{g}^{bc} \partial_u \tilde{g}_{bc}) - \tilde{\nabla}_b (\tilde{g}^{bc} \partial_u \tilde{g}_{ac}) = 0,$$

a harmonic $(p+1)$ -form C in wave-front space

$$\tilde{d}C = \tilde{d}^*C = 0,$$

with arbitrary u -dependence, an arbitrary function $\varphi(u)$; and a function $K(u, x^a)$ satisfying the equation

$$\tilde{\nabla}^2 K + \frac{1}{4}\partial_u \tilde{g}^{ab}\partial_u \tilde{g}_{ab} + \frac{1}{2}\tilde{g}^{ab}\partial_u^2 \tilde{g}_{ab} + \frac{1}{2}(\partial_u \varphi)^2 + \frac{(-1)^{p+1}}{2(p+1)!}e^{-2a\varphi}C^2 = 0.$$

The simplest choice of Ricci-flat wave-front space leads to the solutions

$$\begin{aligned} \tilde{g}_{ab} &= -\delta_{ab}, & C &= C(u), & \varphi &= \varphi(u), \\ K &= H + A, & A &\equiv A_{ab}(u)x^a x^b = -\frac{1}{4}\left[(\partial_u \varphi)^2 + \frac{(-1)^{p+1}}{(p+1)!}e^{-2a\varphi}C^2\right](\text{tr}M)^{-1}M_{ab}x^a x^b, \end{aligned} \quad (1.4)$$

where $H = H(x^a)$ is an arbitrary harmonic function in wave-front space, M_{ab} is a constant symmetric matrix and C and φ are just arbitrary functions of u .

The function K that contains all the information has, therefore, two pieces: the harmonic function $H(x)$, independent of the gauge field and dilaton (i.e. purely gravitational), and the matrix $A_{ab}(u)$ that depends on the gauge field and dilaton. One can argue that H represents excitations over a vacuum that consists of a self-supported (source-less) gauge field and dilaton and a metric described by $A_{ab}(u)$. For instance, one can try to match the above solution with a charged, mass-less, p -brane source with effective action

$$\begin{aligned} S_p[X^\mu, \gamma_{ij}] &= -T_p \int d^{p+1}\xi \sqrt{|\gamma|} e^{-2b\varphi} \gamma^{ij} \partial_i X^\mu \partial_j X^\nu g_{\mu\nu} + \\ &+ \frac{(-1)^{p+1}\mu_p}{(p+1)!} \int d^{p+1}\xi A_{(p+1)\mu_1 \dots \mu_{p+1}} \partial_{i_1} X^{\mu_1} \dots \partial_{i_{p+1}} X^{\mu_{p+1}} \epsilon^{i_1 \dots i_{p+1}}. \end{aligned} \quad (1.5)$$

The following Ansatz⁵

$$U(\xi) = 0, \quad V(\xi) = \alpha \xi^0, \quad X^a(\xi) = 0, \quad \sqrt{|\gamma|} \gamma^{00} = 1, \quad (1.6)$$

α being some constant and ξ^0 being the worldvolume time coordinate (plus the above values Eq. (1.4) for the spacetime fields) representing the brane moving in a direction transverse to its worldvolume reduces all the equations of motion to only one which is not automatically satisfied,⁶ i.e.

$$\partial_a \partial_a H = 16\pi G_N^{(d)} T_p e^{-2b\varphi(0)} \alpha^2 \delta(u) \delta(\vec{x}_8). \quad (1.7)$$

Thus, only H feels the source and the gauge field seems to be self-supported. The solutions with $H = 0$ can be interpreted as vacua and can be described as homogeneous spaces [25, 2] (*Hpp*-waves). Actually, the presence of a covariantly constant null vector

⁵Such an Ansatz was also discussed in Ref. [7] for probing the type IIB's KG wave, but was found to be inconsistent. Here such trouble is avoided because, contrary to Ref. [7], a massless p -brane is used.

⁶For $p = 0$ the charge μ_0 has to be set to zero in order to satisfy the equation of motion for that gauge field, but in the other cases the value of μ_p does not play any role.

ensures that at least half of the supersymmetries will always be unbroken if we embed the above solutions in a supergravity theory (even for $H \neq 0$) but in some cases (the Kowalski-Glikman solutions [1, 3, 19, 23]) there are Hpp-wave solutions that preserve all the supersymmetries. See [26] for a discussion on waves that preserve fractions of the supersymmetry.

It was recently shown [14] that the maximal amount of supersymmetry that can be preserved in a circle compactification of the KG10 solution [3] is 3/4 and the same thing holds for the 11-dimensional KG wave [1]. Although one would expect the same to happen in the $N = 2$ $d = 6, 5, 4$ KG-solutions, we are going to show that they are related by dimensional reduction. First of all, the susy preserving dimensional reduction is possible after a change of coordinates in which the dependence on the compact coordinate is removed at the expense of introducing a non-vanishing Sagnac connection. It turns out that in the new coordinates the Killing spinors are independent of the compact coordinates so that dimensional reduction will preserve all of them. Furthermore, the Sagnac connection becomes a KK vector that combines in the right way with the other vector fields present to cancel the matter multiples that arise in the two dimensional reductions involved.

1.2 Reduction of $KG6(2, 0)$ to $KG5$

$N = (2, 0), d = 6$ supergravity⁷ consists of the metric $\hat{e}^{\hat{a}}_{\hat{\mu}}$, 2-form field $\hat{B}^-_{\hat{\mu}\hat{\nu}}$ with anti-self-dual field strength $\hat{H} = 3\partial\hat{B}^-$ and positive-chirality symplectic Majorana-Weyl gravitino $\hat{\psi}^+_{\hat{\mu}}$. The bosonic equations of motion can be derived from the action

$$\hat{S} = \int d^6\hat{x} \sqrt{|\hat{g}|} [\hat{R} + \frac{1}{12}\hat{H}^2], \quad (1.8)$$

imposing afterward the anti-self-duality constraint $*\hat{H}^- = -\hat{H}^-$. The gravitino supersymmetry transformation rule is (for zero fermions)

$$\delta_{\hat{\epsilon}^+} \hat{\psi}^+_{\hat{a}} = \left(\hat{\nabla}_{\hat{a}} - \frac{1}{48} \hat{H}^- \hat{\gamma}_{\hat{a}} \right) \hat{\epsilon}^+. \quad (1.9)$$

This can be reduced to $N = 2, d = 5$ supergravity (metric e^a_{μ} , *graviphoton* vector field \mathcal{V}_{μ} and symplectic-Majorana gravitino ψ_{μ}) coupled to a vector multiplet consisting of a gaugino (the 6th component of the 6-dimensional gravitino, a real scalar (the KK one) and a vector field \mathcal{W}_{μ} . The vector fields \mathcal{V}_{μ} and \mathcal{W}_{μ} are combinations of scalars, the KK vector field that comes from the 6-dimensional metric A_{μ} and the vector field that comes from the 6-dimensional 2-form B_{μ} . The identification of the right combinations will be made by imposing consistency of the truncation.

Using the same techniques as in the reduction of $N = 2B, d = 10$ supergravity on a circle Ref. [28] one gets the 5-dimensional action

$$S = \int d^5x \sqrt{|g|} k [R - \frac{1}{4}k^2 F^2(A) - \frac{1}{4}k^{-2} F^2(B) + \frac{\epsilon}{8\sqrt{|g|}} k^{-1} F(A)F(B)B]. \quad (1.10)$$

⁷Our conventions are essentially those of Ref. [27] with some changes in the normalizations of the fields. In particular $\gamma_7 = \gamma_0 \cdots \gamma_5$, $\gamma_7^2 = +1$, $\epsilon^{012345} = +1$, $\gamma^{a_1 \cdots a_n} = \frac{(-1)^{[n/2]}}{(6-n)!} \epsilon^{a_1 \cdots a_n b_1 \cdots b_{6-n}} \gamma_{b_1 \cdots b_{6-n}} \gamma_7$. Positive and negative chiralities are defined by $\gamma_7 \hat{\psi}^{\pm} = \pm \hat{\psi}^{\pm}$.

The truncation to pure supergravity involves setting $k = 1$ consistently, i.e. in such a way that its equation of motion is always satisfied. The k equation of motion with $k = 1$ (upon use of Einstein's equation) implies the constraint

$$F^2(B) = 2F^2(A). \quad (1.11)$$

Let us introduce to linear combinations $\mathcal{F}(\mathcal{V}), \mathcal{G}(\mathcal{W})$ of the vector field strengths

$$\begin{cases} \mathcal{F}(\mathcal{V}) &= \alpha F(A) + \beta F(B), \\ \mathcal{G}(\mathcal{W}) &= -\beta F(A) + \alpha F(B), \end{cases} \quad (1.12)$$

with $\alpha^2 + \beta^2 = 1$. Substituting them into the above constraint we see that it is automatically satisfied with $\mathcal{G}(\mathcal{W}) = 0$ and $\beta^2 = 2\alpha^2$, so $\alpha = s(\alpha)/\sqrt{3}$ and $\beta = s(\beta)\sqrt{2}/\sqrt{3}$. These conditions reduce the equations of motion of A and B to a single equation for \mathcal{V} . This equation and the resulting Einstein equation can be derived from the action

$$S = \int d^5x \sqrt{|g|} [R - \frac{1}{4}\mathcal{F}^2 + s(\alpha) \frac{\epsilon}{12\sqrt{3}\sqrt{|g|}} \mathcal{F}\mathcal{F}\mathcal{V}], \quad (1.13)$$

which is that of the bosonic sector of $N = 2, d = 5$ supergravity [29]. The relative sign of α and β will be fixed by supersymmetry: using the decomposition

$$\begin{aligned} \hat{\gamma}^a &= \gamma^a \otimes \sigma^1, & a = 0, 1, 2, 3, 4, \\ \hat{\gamma}^5 &= \mathbb{I} \otimes i\sigma^2, \\ \hat{\gamma}^7 &= \hat{\gamma}_0 \cdots \hat{\gamma}_5 = \mathbb{I} \otimes \sigma^3, \end{aligned} \quad (1.14)$$

where the γ^a s are 5-dimensional gamma matrices satisfying $\gamma_0 \cdots \gamma_4 = \mathbb{I}$, using chirality, we can split the gravitino supersymmetry transformation rule into

$$\begin{cases} \delta_\epsilon \hat{\psi}_a &= \left\{ \nabla_a - \frac{1}{8\sqrt{2}} k^{-1} \mathcal{F}(B) \gamma_a - \frac{1}{4} k \mathcal{F}_a(A) \right\} \epsilon, \\ \delta_\epsilon \hat{\psi}_w &= \left\{ \partial_w + \frac{1}{2} \not{\partial} \log k + \frac{1}{8} k \mathcal{F}(A) - \frac{1}{8\sqrt{2}} k^{-1} \mathcal{F}(B) \right\} \epsilon. \end{cases} \quad (1.15)$$

$\hat{\psi}_w$ is the 5-dimensional gaugino and its supersymmetry transformation has to be identically zero. This can be achieved by taking ϵ independent of w and identifying $s(\alpha) = s(\beta)$ so

$$\mathcal{G} \equiv s(\alpha) \left(\frac{1}{\sqrt{3}} F(B) - \sqrt{\frac{2}{3}} F(A) \right) \equiv 0. \quad (1.16)$$

It only remains the supersymmetry transformation law of $\hat{\psi}_a$ that becomes the 5-dimensional gravitino. Expressed in terms of the surviving vector field, it takes the right form⁸ [29]

⁸Actually, either the sign of the Chern-Simons term or the \mathcal{F} term in the supersymmetry transformation rule in Ref. [29] is wrong. Choosing the sign of α we can make either of them coincide with those in Eqs. (1.10) and (1.17), but not both at the same time. A further check of these signs is provided by the reduction to $d = 4$: the consistency conditions for the truncation to pure $N = 2, d = 4$ supergravity coming from the action and the gaugino supersymmetry transformation rule are incompatible with the signs of Ref. [29] but fully compatible with ours.

$$\delta_\epsilon \psi_a = \left\{ \nabla_a - s(\alpha) \frac{1}{8\sqrt{3}} (\gamma^{bc} \gamma_a + 2\gamma^b g^c_a) \mathcal{F}_{bc} \right\} \epsilon. \quad (1.17)$$

The relation between 6- and 5-dimensional pure supergravity fields is

$$\left| \begin{array}{l} \hat{g}_{\underline{w}\underline{w}} = -1, \\ \hat{g}_{\underline{\mu}\underline{w}} = \frac{s(\alpha)}{\sqrt{3}} \mathcal{V}_\mu, \end{array} \right| \left| \begin{array}{l} \hat{B}_{\underline{\mu}\underline{w}}^- = \frac{s(\alpha)}{\sqrt{3}} \mathcal{V}_\mu, \\ \hat{g}_{\underline{\mu}\underline{\nu}} = g_{\mu\nu} - \frac{1}{3} \mathcal{V}_\mu \mathcal{V}_\nu, \end{array} \right| \quad (1.18)$$

while the $\hat{B}_{\underline{\mu}\underline{\nu}}^-$ components can be found imposing anti-self-duality.

Now, let us consider the $KG6(2, 0)$ solution [19] in canonical coordinates with \hat{B}^- in a convenient gauge

$$KG6(2, 0) : \left\{ \begin{array}{l} d\hat{s}^2 = 2du[dv + \frac{\lambda_6^2}{8} \vec{x}_{(4)}^2 du] - d\vec{x}_{(4)}^2, \quad \vec{x}_{(4)} \equiv (x, y, z, w), \\ \hat{B}^- = \lambda_6 du \wedge (zdw - xdy), \\ \hat{\epsilon} = \left[1 - \frac{\lambda_6}{4} \hat{\gamma}^{+23} \vec{x}_{(4)} \cdot \hat{\vec{\gamma}} \right] \exp\left(\frac{u\lambda_6}{4} \hat{\gamma}^{+23} \hat{\gamma}^-\right) \hat{\epsilon}^{(0)}, \end{array} \right. \quad (1.19)$$

Performing the coordinate transformations

$$\left\{ \begin{array}{l} z = \cos\left(\frac{\lambda_6}{2}u\right)z' + \sin\left(\frac{\lambda_6}{2}u\right)w', \\ w = -\sin\left(\frac{\lambda_6}{2}u\right)z' + \cos\left(\frac{\lambda_6}{2}u\right)w', \\ v = v' + \frac{\lambda_6}{2}z'w', \end{array} \right. \quad (1.20)$$

the solution takes the w' -independent form

$$KG6(2, 0) : \left\{ \begin{array}{l} \hat{s}^2 = 2du[dv + \frac{\lambda_6^2}{8}(x^2 + y^2)du + \lambda_6 z' dw'] - d\vec{x}'_{(4)}{}^2, \quad \vec{x}'_{(4)} \equiv (x, y, z', w'), \\ \hat{B}^- = \lambda_6 du \wedge (z'dw' - xdy), \\ \hat{\epsilon} = \left[1 + \frac{\lambda_6}{4} \hat{\gamma}^{+23} \{x\hat{\gamma}_2 + y\hat{\gamma}_3\} \right] \exp\left(\frac{u\lambda_6}{4} \{\hat{\gamma}^{45} + \hat{\gamma}^{+23} \hat{\gamma}^-\}\right) \hat{\epsilon}^{(0)}. \end{array} \right. \quad (1.21)$$

It is easy to see that it satisfies the truncation conditions

$$\hat{g}_{\underline{w}\underline{w}} = -1, \quad \hat{B}_{\underline{\mu}\underline{w}}^- = \hat{g}_{\underline{\mu}\underline{w}}, \quad \partial_{\underline{w}} \hat{\epsilon}^+ = 0, \quad (1.22)$$

and, thus, it can be reduced to a solution of pure $N = 2, d = 5$ supergravity that turns out to be the maximally supersymmetric $KG5$ solution [28]:

$$KG5 : \left\{ \begin{array}{l} ds^2 = 2du[dv + \frac{\lambda_5^2}{24}(4z'^2 + x^2 + y^2)du] - d\vec{x}'_{(3)}{}^2, \quad \vec{x}'_{(3)} \equiv (x, y, z'), \\ \mathcal{F} = \lambda_5 du \wedge dz', \quad \lambda_5 = -s(\alpha)\sqrt{3}\lambda_6. \end{array} \right. \quad (1.23)$$

1.3 Reduction of $KG5$ to $KG4$

The action Eq. (1.10) can be straightforwardly reduced to $d = 5$ dimensions giving the action of $N = 2, d = 4$ supergravity (consisting of the metric, the *graviphoton* vector field V_μ and a gravitino) coupled to a vector multiplet (consisting of a vector W_μ and two real scalars k, l plus a gaugino) [30]. The two vectors will be combinations of the KK vector A_μ that comes from the metric and the vector B_μ that comes from the 5-dimensional vector \mathcal{V}_μ . To determine the right combinations, we study the consistency of the truncation of the fields that belong for sure to the matter multiplet $k = 1, l = 0$ and the gaugino.

The action for the 4-dimensional bosonic fields is

$$S = \int d^4x \sqrt{|g|} k \left\{ R + \frac{1}{2} k^{-2} (\partial l)^2 - \frac{1}{4} k^2 F^2(A) - \frac{1}{4} [F(B) + l F(A)]^2 \right. \\ \left. + s(\alpha) \frac{k^{-1} l}{4\sqrt{3}\sqrt{|g|}} \epsilon [F(B) + l F(A) - 2A\partial l]^2 \right\}. \quad (1.24)$$

Setting $k = 1, l = 0$ in the equations of motion of k and l we get two constraints:

$$\begin{cases} 3F^2(A) + F^2(B) = 0, \\ \sqrt{3}F(A) - s(\alpha)^* F(B) = 0. \end{cases} \quad (1.25)$$

The second constraint implies the first and is actually sufficient to identify the graviphoton and the matter vector field strengths⁹ up to global, irrelevant, signs, that we fix arbitrarily

$$\begin{cases} F(V) = \frac{1}{2}^* F(A) - s(\alpha) \frac{\sqrt{3}}{2} F(B), \\ F(W) = -\frac{\sqrt{3}}{2}^* F(A) - s(\alpha) \frac{1}{2} F(B). \end{cases} \quad (1.26)$$

Setting $F(W) = 0$ (which is consistent with the W_μ equation of motion) we get the action of (the bosonic sector of) pure $N = 2, d = 4$ supergravity (the Einstein-Maxwell action)

$$S = \int d^4x \sqrt{|g|} [R - \frac{1}{4} F^2(V)]. \quad (1.27)$$

We can see that this truncation is consistent with the supersymmetry transformation rules. The 5-dimensional matrices $\hat{\gamma}^{\hat{a}}$ decompose into 4-dimensional matrices as follows:

$$\hat{\gamma}^a = \gamma^a, \quad a = 0, 1, 2, 3, \quad \hat{\gamma}^4 = -i\gamma_5 = \gamma_0\gamma_1\gamma_2\gamma_3. \quad (1.28)$$

The 5-dimensional symplectic-Majorana spinors are a pair of ordinary 4-component Dirac spinors related by the symplectic-Majorana constraint. Thus, in $d = 4$ we simply keep one of them, which will be unconstrained and decomposable, if necessary, into a pair of 4-dimensional Majorana spinors.

Now if the supersymmetry parameter is independent of the compactification direction y and we set $k = 1, l = 0$ in the y component of the gravitino transformation rule (which should become the gaugino transformation rule), we find that

⁹For $k = 1, l = 0$ only.

$$\delta_\epsilon \hat{\psi}_y = \frac{i}{4\sqrt{3}} \mathcal{F}(W) \epsilon. \quad (1.29)$$

and, so, the truncation $F(W) = 0$ is consistent with setting the gaugino to zero. The supersymmetry transformation rule of the surviving gravitino is

$$\delta_\epsilon \psi_a = \left[\nabla_a + \frac{1}{8} \mathcal{F}(V) \gamma_a \right] \epsilon. \quad (1.30)$$

The relation between 5-dimensional and 4-dimensional fields that satisfy the truncation condition is

$$\left| \begin{array}{l} \hat{g}_{\underline{y}\underline{y}} = -1, \\ 2\partial_{[\mu} \hat{g}_{\nu]\underline{y}} = \frac{1}{4\sqrt{|g|}} \epsilon_{\mu\nu\rho\sigma} F^{\rho\sigma}(V), \\ \hat{g}_{\mu\nu} = g_{\mu\nu} - \hat{g}_{\underline{\mu}\underline{\nu}} \hat{g}_{\underline{y}\underline{y}}, \end{array} \right| \quad \left| \begin{array}{l} \hat{\mathcal{V}}_{\underline{y}} = 0, \\ \hat{\mathcal{V}}_{\underline{\mu}} = -s(\alpha) \frac{\sqrt{3}}{2} V_{\underline{\mu}}, \end{array} \right. \quad (1.31)$$

Now, to apply these results to the $KG5$ solution Eq. (1.23) we first perform the change of coordinates

$$\left\{ \begin{array}{l} x = \cos\left(\frac{\lambda_5}{2\sqrt{3}}u\right)x' + \sin\left(\frac{\lambda_5}{2\sqrt{3}}u\right)y', \\ y = -\sin\left(\frac{\lambda_5}{2\sqrt{3}}u\right)x' + \cos\left(\frac{\lambda_5}{2\sqrt{3}}u\right)y', \\ v' = v'' + \frac{\lambda_5}{2\sqrt{3}}x'y', \end{array} \right. \quad (1.32)$$

that puts the $KG5$ solution in the y' -independent form

$$KG5: \quad \left\{ \begin{array}{l} ds^2 = 2du[dv'' + \frac{\lambda_5^2}{6}z'^2 du + \frac{\lambda_5}{\sqrt{3}}x'dy'] - d\vec{x}'_{(3)}{}^2, \quad \vec{x}'_{(3)} \equiv (x', y', z'), \\ \mathcal{F} = \lambda_5 du \wedge dz'. \end{array} \right. \quad (1.33)$$

In this form, the $KG5$ solution just happens to satisfy the truncation condition that allows us to reduce it to a pure $N = 2, d = 4$ supergravity solutions that turns out to be the $KG4$ maximally supersymmetric spacetime [23], as promised

$$KG4: \quad \left\{ \begin{array}{l} ds^2 = 2du[dv + \frac{\lambda_4^2}{8}\vec{x}'_{(2)}{}^2 du] - d\vec{x}'_{(2)}{}^2, \quad \vec{x}'_{(2)} \equiv (x', z'), \\ F = \lambda_4 du \wedge dz', \quad \lambda_4 = s(\alpha) \frac{2}{\sqrt{3}} \lambda_5. \end{array} \right. \quad (1.34)$$

At first sight it is surprising that in all cases the truncation condition can be satisfied, at least in a certain gauge. Actually, it is easy to see that it must happen by thinking in terms of oxidation of the lower-dimensional solutions: Since the $N = 2, d = 5$ theory can be reduced to $N = 2, d = 4$ supergravity coupled to a vector multiplet that can be consistently truncated, any solution of pure $N = 2, d = 4$ supergravity can be uplifted to a solution of the $N = 2, d = 5$ theory with the same, or bigger, amount of supersymmetry. Therefore, the $KG4$ solution can be uplifted to a maximally supersymmetric solution of the

$N = 2, d = 5$ theory which turns out to be the $KG5$ solution in non-canonical coordinates. Essentially the same mechanism works in the oxidation of the $KG5$ solution to a maximally supersymmetric solution of $N = (2, 0), d = 6$ that turns out to be the $KG6(2, 0)$.

Now it is clear that the same should happen in all cases: all solutions of pure $N = 2, d = 4$ supergravity must be related via dimensional reduction/oxidation to pure $N = 2, d = 5$ and $N = (2, 0), d = 6$ supergravity solutions that preserve the same amount of supersymmetry. In particular, maximally supersymmetric solutions of these three theories should be related. We have seen that this is true for the KG spacetimes and now we are going to study the $aDS_n \times S^m$ spacetimes.

2 Oxidation of Maximally Supersymmetric $d = 4, 5, 6$ $aDS_n \times S^m$ Spacetimes

2.1 Oxidation of the Robinson-Bertotti Solution

The Robinson-Bertotti solution [24] can be obtained either as a particular member of the Majumdar-Papapetrou family of solutions of the Einstein-Maxwell equations [31] or as the near-horizon limit of the extreme Reissner-Nordström black hole solution [32] and is given in its electric and magnetic versions by

$$\begin{cases} ds^2 &= R_2^2 d\Pi_{(2)}^2 - R_2^2 d\Omega_{(2)}^2, \\ F_{\chi\phi} &= -2R_2 \text{ch}\chi, \\ F_{\theta\varphi} &= 2R_2 \sin\theta, \end{cases} \quad (2.1)$$

with

$$\begin{cases} d\Pi_{(2)}^2 &\equiv \text{ch}^2\chi d\phi^2 - d\chi^2, \\ d\Omega_{(2)}^2 &\equiv d\theta^2 + \sin^2\theta d\varphi^2, \end{cases} \quad (2.2)$$

The metric is that of the direct product of that of aDS_2 with radius R_2 in global coordinates $\phi \in [0, 2\pi), \chi \in [0, \infty)$ and that of S^2 with radius R_2 in standard spherical coordinates $\theta \in [0, \pi], \varphi \in [0, 2\pi)$. It is known to be maximally supersymmetric in $N = 2, d = 4$ supergravity [32, 33] in both the electric and magnetic cases, since the whole $N = 2, d = 4$ supergravity is invariant under chiral/dual transformations.

2.1.1 Electric Case

Following the rules found in the previous section (with $s(\alpha) = +1$) and, further, *assuming that the compact coordinate $y \in [0, 4\pi R_2)$ and using instead $\psi = y/R_2$* , we find the $d=5$ solution

$$\begin{cases} d\hat{s}^2 &= R_2^2 d\Pi_{(2)}^2 - (2R_2)^2 d\Omega_{(3)}^2, \\ \hat{\mathcal{F}}_{\chi\phi} &= \sqrt{3}R_2 \text{ch}\chi, \end{cases} \quad (2.3)$$

with

$$d\Omega_{(3)}^2 \equiv \frac{1}{4} [d\Omega_{(2)}^2 + (d\psi + \cos\theta d\varphi)^2] , \quad (2.4)$$

which is the direct product of aDS_2 with radius R_2 in global coordinates $\phi \in [0, 2\pi)$, $\chi \in [0, \infty)$ and that of S^3 with radius $2R_2$ in Euler-angle coordinates $\theta \in [0, \pi]$, $\varphi \in [0, 2\pi)$, $\psi \in [0, 4\pi)$. This solution is the near-horizon limit of a 5-dimensional extreme black hole and it is maximally supersymmetric [34].

2.1.2 Magnetic Case

Straightforward application of the oxidation rules, now with the compact coordinate in the range $y \in [2, 2\pi R_2)$ with $\eta = y/R_2$ and rescaling $\chi \rightarrow \chi/2$ leads us to

$$\begin{cases} d\hat{s}^2 &= (2R_2)^2 d\Pi_{(3)}^2 - R_2^2 d\Omega_{(2)}^2 , \\ \hat{\mathcal{F}}_{\theta\varphi} &= -\sqrt{3}R_2 \sin\theta , \end{cases} \quad (2.5)$$

with

$$d\Pi_{(3)}^2 \equiv \frac{1}{4} [d\Pi_{(2)}^2 - (d\psi + \text{sh}(\chi/2)d\phi)^2] , \quad (2.6)$$

the metric of aDS_3 in a form that suggests that this spacetime can be understood as an S^1 fibration over aDS_2 .

The above 5-dimensional solution has the metric of $aDS_3 \times S^2$ which is the near-horizon limit of the extreme $d = 5$ string [20].

It has been observed many times that $N = 2, d = 5$ supergravity (its action and field content) is a theory that resembles very much $N = 1, d = 11$ supergravity [29]. One additional similarity is the presence of two maximally supersymmetric vacua ($aDS_4 \times S^7$ and $aDS_7 \times S^4$) which are respectively the near-horizon limits of the solutions that describe the extended objects of the theory: black hole and string in $d = 5$ and M2 and M5 branes in $d = 11$. However, it should be clear that we can obtain new $d = 5$ vacua from new $d = 4$ vacua, should they exist. As a matter of fact they do exist: the dyonic RB solutions which have both electric and magnetic components of the electromagnetic field and share the same $aDS_2 \times S^2$ metric.

2.1.3 Dyonic Case

The dyonic RB solution is given by

$$\begin{cases} ds^2 &= R_2^2 d\Pi_{(2)}^2 - R_2^2 d\Omega_{(2)}^2 , \\ F &= -\frac{2}{R_2} \cos\xi dr \wedge dt + 2R_2 \sin\xi \sin\theta d\theta \wedge d\varphi , \end{cases} \quad (2.7)$$

where now, for convenience, we use the following aDS_2 metric:

$$R_2^2 d\Pi_{(2)}^2 = \left(\frac{r}{R_2}\right)^2 dt^2 - \left(\frac{R_2}{r}\right)^2 dr^2 . \quad (2.8)$$

This family of solutions, that includes the purely electric and magnetic cases that we have just seen, has another parameter apart from the radius R_2 : the duality rotation angle ξ .

Following the oxidation rules we find a 5-dimensional family of maximally supersymmetric solutions

$$\left\{ \begin{array}{l} d\hat{s}^2 = \left(\frac{r}{R_2} \right)^2 dt^2 - \left(\frac{R_2}{r} \right)^2 dr^2 - \left(dy - \frac{r}{R_2} \sin \xi dt + R_2 \cos \xi \cos \theta d\varphi \right)^2 - R_2^2 d\Omega_{(2)}^2, \\ \hat{\mathcal{F}} = \frac{\sqrt{3}}{R_2} \cos \xi dr \wedge dt - \sqrt{3} R_2 \sin \xi \sin \theta d\theta \wedge d\varphi, \end{array} \right. \quad (2.9)$$

The explicit form of the Killing spinors in this case reads

$$\epsilon = \exp(-X \log(r)) \exp(t Y) \exp(\theta Z) \exp\left(-\frac{\varphi}{2} \gamma^{34}\right) \epsilon_{(0)}, \quad (2.10)$$

where

$$X = \frac{1}{2} [\sin(\xi) \gamma^{02} + \cos(\xi) \gamma^0], \quad (2.11)$$

$$Y = \frac{1}{2} [\sin(\xi) \gamma^{12} + \cos(\xi) \gamma^1 - \gamma^{01}], \quad (2.12)$$

$$Z = \frac{1}{2} [\cos(\xi) \gamma^{24} + \sin(\xi) \gamma^4], \quad (2.13)$$

After the coordinate redefinitions

$$\cos(\xi) t \rightarrow t, \quad \frac{y}{R_2 \cos(\xi)} \rightarrow \psi, \quad (2.14)$$

takes the form

$$\left\{ \begin{array}{l} d\hat{s}^2 = \left[\frac{r}{R_2} dt + R_2 \sin \xi (d\psi + \cos \theta d\varphi) \right]^2 - \left(\frac{R_2}{r} \right)^2 dr^2 - (2R_2)^2 d\Omega_{(3)}^2, \\ \hat{\mathcal{F}} = \frac{\sqrt{3}}{R_2} dr \wedge dt - \sqrt{3} R_2 \sin \xi \sin \theta d\theta \wedge d\varphi. \end{array} \right. \quad (2.15)$$

If we set $R_2 = 1/2$, $\sin \xi = j$, $2t \rightarrow t$ and $r \rightarrow r^2$ we recover a solution that describes the near-horizon limit of the supersymmetric [21] rotating $d = 5$ black hole, given in [22, 35]. While it was known that in the zero-rotation limit $j = 0$ this solution has the metric of $aDS_2 \times S^3$, the result in the limiting case $j \rightarrow 1$ was unknown since it is a singular limit. However, by means of the inverse of the above coordinate transformations, the limit can be taken in such a way that the limiting metric, at $\xi = \pi/2$, is regular: $aDS_3 \times S^2$. Thus, the near-horizon limit of the $j = 1$ supersymmetric rotating black hole and the near-horizon limit of the string are identical.

Finally, let us mention the superalgebras associated to these vacua. As was pointed out in Ref. [22], the superalgebra associated to the solution (2.15), is $su(1, 1|2) \oplus u(1)$ when $0 < j < 1$ and gets enhanced to $su(1, 1|2) \oplus su(2)$ when $j = 0$. Combining this with the smooth $\xi = \pi/2$ limit for the family (2.9), one sees that the superalgebra associated to the $aDS_3 \times S^2$ has to be $su(1, 1|2) \oplus sl(2, \mathbb{R})$ and not $sl(2, \mathbb{R}) \ltimes su(1, 1|2)$ as was hinted at in Ref. [36].

2.2 Oxidation to $d = 6$

The oxidation of Eq. (2.9) gives, after rotation of the two isometric coordinates y, w by the angle associated to the 4-dimensional electric-magnetic duality ξ

$$\begin{cases} w &= \cos \xi \eta + R_2 \sin \xi \psi, \\ y &= -\sin \xi \eta + R_2 \cos \xi \psi, \end{cases} \quad (2.16)$$

one recovers the solution

$$\begin{cases} d\hat{s}^2 &= (2R_2)^2 d\Pi_{(3)}^2 - (2R_2)^2 d\Omega_{(3)}^2, \\ \hat{B}^- &= \frac{r}{R_2} d\eta \wedge dt - R_2^2 \cos \theta d\varphi \wedge d\psi, \end{cases} \quad (2.17)$$

whose metric is that of $aDS_3 \times S^3$, the maximally supersymmetric solution which is the near-horizons limit of a self-dual string [20]. It is known that the uplifting of the near-horizon limit of the rotating $d = 5$ black hole gives, for any value of the rotation parameter, $aDS_3 \times S^3$ [37].

3 Conclusions

In this article we have shown that the known supersymmetric vacua of the $d = 6$ $N = (2, 0)$, $d = 5$ $N = 2$ and $d = 4$ $N = 2$ supergravity are linked by dimensional reduction. Although this may come as a bit of a surprise when thinking in terms of dimensional reduction, it is quite obvious from the oxidation point of view: since all three theories have 8 supercharges and oxidation cannot reduce the number of preserved supersymmetries, a lower dimensional maximally supersymmetric solution must lift to a maximally supersymmetric solution.

From the supergravity point of view, the relations can hold because the dimensionally reduced theories can be truncated consistently to the minimal $N = 2$ supergravity, *i.e.* without any matter couplings. A subtle point in the dimensional reduction is that for the Killing spinors to survive the dimensional reduction, the Killing spinors must be independent of the compact coordinates. In a coordinate independent way, this means that there must be a Killing vector whose action on the Killing spinor vanishes, or put differently, there is a bosonic generator in the superalgebra associated to the solution [38], that is represented trivially on the supercharges. Actually, it is not difficult to see that from the superalgebra point of view, the relation between the $N = 2$ vacua was going to hold.

For definiteness let us consider the superalgebras associated to the $aDS_p \times S^q$ spacetimes (See table (1)), the analogous results for the KG-waves being obtainable by a Wigner-Inönü contraction on their $aDS \times S$ counterparts [39].¹⁰ It is clear that the way to preserve supersymmetry is by embedding the generator of translations in the compactification direction, in the non- $su(1, 1|2)$ part of the superalgebra. For the dimensional reduction from $d = 6$ to $d = 5$, there are basically 3 choices, corresponding to the 3 5-dimensional solutions given in Eqs. (2.5, 2.5, 2.9). For a further reduction to $d = 4$ there is basically one way to embed such a translation generator. Note that the chain of relations exposed in this letter is quite unique among the vacua: Had we considered for example the $aDS_3 \times S^3$ solution in the

¹⁰The exception is of course the family of metrics in Eq. (2.9), when $\xi \neq 0, \pi/2$, since its Penrose contraction has 2 more isometries.

Space	Theory	Solution	Superalgebra
$aDS_3 \times S^3$	$N = (2, 0) \ d = 6$	(2.17)	$su(1, 1 2) \oplus sl(2, \mathbb{R}) \oplus su(2)$
$aDS_3 \times S^2$	$N = 2 \ d = 5$	(2.5)	$su(1, 1 2) \oplus sl(2, \mathbb{R})$
$aDS_2 \times S^3$	$N = 2 \ d = 5$	(2.3)	$su(1, 1 2) \oplus su(2)$
Dyonic	$N = 2 \ d = 5$	(2.9)	$su(1, 1 2) \oplus u(1)$
$aDS_2 \times S^2$	$N = 2 \ d = 4$	(2.1)	$su(1, 1 2)$

Table 1: Solutions and their associated superalgebra.

$d = 6 \ N = (4, 0)$ supergravity, we would have had to conclude that, since the associated superalgebra is $su(1, 1|2) \oplus su(1, 1|2)$, there is no way to preserve the 16 supercharges in a circle compactification.

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